Sum and Difference Identities for Sine and Tangent

There are also sum and difference identities for sine and tangent:

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$
$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Mnemonic for sine sum/difference: "Sine can't change signs."

Examples:

Find the exact value of $\sin(75^\circ)$.

Find the exact value of $\sin(\pi/12)$.

Find the exact value of $\tan(15^\circ)$.

Find the exact value of $\tan(5\pi/12)$.

Find the exact value of $\sin(\alpha+\beta)$ if $\sin\alpha=-3/5$ and $\cos\beta=-1/3$ with α in Quadrant IV and β in Quadrant III.